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Sensitivity analysis associated with the evaluation of measurement uncertainty: application to a computational code in metrology

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Abstract

In metrology, Monte-Carlo Methods are used to evaluate the measurement uncertainty whereas metrologists were used to apply a Taylor series approximation and to perform a local sensitivity analysis with the partial derivatives. With a Monte-Carlo method, a global sensitivity approach is considered. The application is a computational code for the propagation of a primary fire source in a room. It consists in the evaluation of measurement uncertainty in a security context, surrounded by a strong regulation. The set of input quantities to consider has been determined by a previous Morris's design. Other sensitivity analysis techniques, such as the SRRC, the Local Polynomial Smoothers (or the FAST), not yet considered in metrology, are used to prioritize the influent input quantities.

Keywords : SRRC, Local Polynomial Sensitivity Indices, Measurement uncertainty, Monte Carlo Method

1. The evaluation of uncertainty of measurement for a computational code

The evaluation of uncertainty in metrology is typically based on the relationship between the quantity of interest Y and the input quantities X_1, \dots, X_n :

$$y = f(x_1, \dots, x_n) \quad (1)$$

The Guide for the evaluation of Uncertainty of Measurement (GUM) describes how to evaluate uncertainty with a Taylor approximation of the measurement model f . Then, sensitivity coefficients are local sensitivity coefficients, they are taken as the partial derivatives of the measurement model according to each input quantity (Allard and Fischer, 2008). With a Monte-Carlo method, many tools for sensitivity analysis may be implemented. Until now, only the One-at-A-Time sensitivity indices are used in the context of measurement uncertainty. Some other tools are available. In this paper, we compare the different indices regarding their computational time, their convergence and their limitations.

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In this practical case, the relationship f , as defined in equation (1), is a computational code called CFAST, associated with a pre and post-treatment Excel sheet. The quantities of interest are the maximal upper and lower layer temperature and the minimal layer height, that is to say three parameters, that may be lethal beyond a given threshold, therefore subject to a strong regulation. The algorithm computes the output quantities from a set of input quantities that includes the size of the room (height, floor area), the global size of the openings (height, width), the properties of the primary fire source (area, rate of heat release) and the inside and outside temperatures. The computational time for a single model run may reach one minute and one hundred and fifty thousand runs are required for the analysis, thus it's important in this case to provide a sensitivity index that doesn't need too many evaluations of the model.

The measurement uncertainty associated to a result has to be as small as possible but also has to remain accurate. Thus, it is important to identify the input quantities that are the most influent with a two-step procedure. First, non-influent input quantities are determined with a Morris's method. Second, a prioritization of the input quantities is built by the different sensitivity indices. Then, it will be interesting to try to improve the measurement process, by reducing the uncertainty of the most important input quantity. In this paper, several indices are compared in order to find the one that is the more efficient to evaluate sensitivity in this practical case, in terms of interpretation, convergence and order of the sensitivity indices.

2. Tools for sensitivity analysis

In the first place, the Standardized Rank Regression Coefficient sensitivity indices have been computed in order to have an estimation of the first order sensitivity indices. It's often a suitable index in the field of measurement uncertainty because the measurement relationship is often linear, or at least monotonic.

In the second place, a variance-based estimator is computed in order to analyze whether there are some higher-order effects involved or not. Particularly, we want to estimate second-order sensitivity indices. The method of Sobol is not suitable in this case from a computational time point of view. FAST (Fourier Amplitude Sensitivity Test) may be considered (Saltelli and al., 2000). Also, a sensitivity index based on the estimation of Local Polynomial Smoothers (LPS) is performed (Da Veiga and al., 2009).

3. Conclusion

The Local Polynomial Smoothers appears to be a good tool for sensitivity analysis. Its computational time is quite reasonable, which enables us to repeat its implementation in order to estimate a standard deviation of the indices and to control their convergence. Moreover, it enables us to compute second-order indices, whereas FAST only computes first-order and total order sensitivity indices.

4. References

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