

Sixth International Conference on Sensitivity Analysis of Model Output

Total Order Reliability Importance in Space PSA

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Abstract

Recent literature has recognized the importance of including interactions in reliability importance measures. However, a study of when interactions do matter in complex PSA models and how they relate with uncertainty has not been performed yet. It is the purpose of this work to investigate such relationship. In order to perform our investigation, we make use of the recently introduced total order reliability importance measures. Applications are obtained via a space PSA model.

Keywords: Space PSA; Sensitivity Analysis; Importance Measures

1. Introduction

When dealing with complex systems, knowing whether system response is determined by individual components or by their interactions is essential. Neglecting interactions in an application in which they do matter could lead to non-completely informed decisions.

Probabilistic Safety Assessment (PSA) models are realistic descriptions of systems behaviour. They provide analysts with a thorough characterization of system failure modes and accident sequences, through the conjunction of event trees and fault trees [Smith(1998); Papazoglou(1998)]. Complex systems as nuclear power plants or space missions are made of a huge number of components, and the resulting PSA models contain hundreds of basic events and thousands of minimal cut sets.

Nonetheless, the multilinear structure of the risk metric makes the presence of interactions certain [Borgonovo (2009)]. However, in a system with n components there are, in principle, $2^n - 1$ interactions. When n is large, computing all interactions becomes readily unmanageable. Hence, the problem faced by analysts is to determine whether interactions matter in the application at hand, while, at the same time, minimizing the computational burden of the analysis. One notes that commonly utilized PRA importance measures do not allow one to evidence the strength and relevance of interactions. For example, the risk achievement worth (RAW) importance measure ranks components based on the effect of their individual failures. The differential importance measure (D) [Borgonovo and Apostolakis (2001)], accounts for individual contributions to small changes. Recent literature has then extended D to include interactions [Zio and Podofillini (2006).]

In this work, we offer a thorough study of the quantitative impact of interactions in determining model response in PSA. The work is based on results in Borgonovo and Smith (2010). We proceed as follows. We make use of the total order differential importance measure (D^T) [Borgonovo (2009)].

$$D_l^T := \frac{\Delta^T G_l}{\Delta G} = \frac{B_l \Delta x_l + \sum_{k=2}^T \sum_{\substack{i_1 < i_2, \dots, i_k \\ l \in i_1 < i_2, \dots, i_k}} J_{i_1, i_2, \dots, i_k}^k(x^0) \prod_{s=1}^k \Delta x_{i_s}}{\sum_{i=1}^N B_i(x^0) \cdot \Delta x_i + \sum_{k=2}^T \sum_{i_1 < i_2, \dots, i_k} J_{i_1, i_2, \dots, i_k}^k(x^0) \prod_{s=1}^k \Delta x_{i_s}} \quad (1)$$

In eq. (1), B_l is the Birnbaum importance of component or basic event l and J_{i_1, i_2, \dots, i_k} is the joint reliability importance of components i_1, i_2, \dots, i_k .

Thus, D_l^T synthesizes in one indicator the Birnbaum importance and the joint reliability importance of all orders related to component l . We note that it is applied to a PSA model for the first time in Borgonovo and Smith (2010). In such work, the following results are obtained:

- All interactions involving more than one initiating events are null;
- Under the rare event approximation, the joint reliability importance of a minimal cut set equals the sum of IE frequencies involving the given minimal cut set. Also, there is no interaction of order higher than the order of the largest minimal cut set plus 1.

We then observe that the PSA risk-metric is smooth function of the basic event probabilities and initiating event frequencies. Thus, for small changes, model response is governed by the first order differential. Since the first order differential is additive, interactions are not felt. Hence, while interactions are present in general, they might not be crucial in the application at hand. One needs to determine "when small is small enough [Borgonovo and Apostolakis (2001)]." We present a method for identifying at what size of the changes interactions cannot be omitted from the analysis. As we are to see, the method also allows us also to relate interactions and epistemic uncertainty.

We then assess the problem of determining interactions at the system level. To do so, we extend D^T to include groups of basic events. We show that, by exploiting the relationship between D^T and the finite change sensitivity indices of Borgonovo (2010), it is possible to introduce total order group sensitivity measures.

We conduct a numerical analysis by applying our findings to a full-fledged PSA model. The PSA model describes the safety analysis of a hypothetical space mission. The model is encoded in the SAPHIRE risk analysis software and contains 382 basic events. We let the model undergo several numerical experiments and determine the conditions under which interactions matter in the analysis. Also, results for the interactions at the level of space mission systems are obtained.

2. References

Borgonovo E. and Apostolakis G.E., 2001: A New Importance Measure for Risk-Informed Decision-Making, *Reliability Engineering and System Safety*, 72 (2), pp. 193-212.

Borgonovo, E., 2009: The reliability importance of components and prime implicants in coherent and non-coherent systems including total-order interactions, *European Journal of Operational Research*, doi:10.1016/j.ejor.2009.10.021.

Borgonovo E., 2010: Sensitivity analysis with finite changes: An application to modified EOQ models, *European Journal of Operational Research*, 200, pp. 127–138.

Borgonovo E. And Smith C.L., 2010: "A Study of Interactions in Complex Safety Assessment: an Application to Space PSA", submitted.

Smith, C. L., "Calculating Conditional Core Damage Probabilities for Nuclear Power Plant Operations" *Reliability Engineering and System Safety*, 59 (1998) 299-307.

Papazoglou I.A., 1998: "Mathematical Foundations for Event Trees," *Reliability Engineering and System Safety*, 61, pp. 169–183.

Zio E. and Podofillini L., 2006: "Accounting for components interactions in the differential importance measure", *Reliability Engineering and System Safety*, 91, pp. 1163–1174.