

Sixth International Conference on Sensitivity Analysis of Model Output

Regression sensitivity analysis for cash flow simulation based real option valuation

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Abstract

Sensitivity analysis on financial options considers how the solution changes because of a change in one of the key parameters (underlying asset value, volatility, exercise price, interest rate, time to maturity, dividends). In case of real option valuation with cash flow simulation, however, these are mostly indirect variables which are computed based on the uncertain direct variables – e.g. demand, unit selling price, and unit costs – in the cash flow calculation. The method presented detects the most significant primary variables, and based on this analysis, shows how changes in the direct uncertainties can be used to estimate with the response surface method the simultaneous changes in the indirect parameters defining the underlying asset process and thus the real option value.

Keywords: sensitivity analysis, cash flow simulation, real options

1. Main text

Real option valuation is an approach to valuing managerial flexibility in investment decisions under uncertainty. These decisions can be related to deferring, extending, postponing or abandoning the project (Trigeorgis 1996). According to the real option thinking and valuation, investment decision can be interpreted as being an option characterized by sequential, irreversible investments made under the conditions of uncertainty with optimal timing (Dixit and Pindyck, 1995). The expression real options stems from utilizing and adapting the mathematics commonly used in valuing financial options related to some uncertain real investments.

Traditional sensitivity analysis on financial options is a local analysis that considers how the solution changes as a result of a small change in one of the parameters of underlying asset value (delta), volatility (vega), exercise price, time to maturity (theta), and interest (rho). With financial options, these aspects can be computed using well-known equations and available market information, allowing precise state-space structure for different underlying asset values for each time period. Parameter values required for real option valuation, however, are usually based more on subjective estimates. Monte Carlo simulation on future cash flows is often used to estimate the underlying asset value, volatility and other possible stochastic process parameters over time. This estimation is highly subjective in

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comparison with the financial option underlying asset that is observable from market data. Also, the parameters affecting the calculation of the Greeks are not the true sources of uncertainty, but rather indirect or secondary parameters that are derived from the primary or direct uncertainty parameters. These primary sources of uncertainty that actually change are those parameters of the cash flow model, e.g. product demand, sales unit price, and variable unit costs. Typically a change in a primary parameter also changes simultaneously several indirect or secondary real option valuation parameters. Therefore, the sensitivity analysis of real options should take this into account.

Sampling based sensitivity analysis is a natural choice for a Monte Carlo cash flow simulation based real option valuation. The method presented in this paper uses regression coefficients as a primary sensitivity analysis method. The procedure shows the behavior of the consolidated stochastic process and how different cash flow components affect it. Based on that, a forward-looking response surface method estimator for the underlying asset and its stochastic process parameters can be constructed. These estimators enable analyzing how much a change in a direct variable changes real option value via changing indirect variables.

After the cash flow simulation, the regression coefficients are calculated and then squared and scaled to unity so that their contribution to overall uncertainty is easier to interpret. Then, the grouping of the factors is done so that the parameters discretized over the time in cash flow calculation are recognized as a group of variables. Then the results can be presented as a contribution to variance tornado chart and a two-dimensional matrix table of primary parameters and time periods. The table shows how much of the total variance can be explained by each primary variable when grouped together, and how much of the variability is related to each time period. Both of these are relevant information considering real options valuation, showing the most significant sources of uncertainty in the cash flow model and how much the uncertainty in terms of variance changes between different time periods. Another important aspect is to recognize the pattern inside a certain group of variables, because some uncertainties often remain sufficiently constant, while some others change significantly over time.

After that, other diagnostic methods are applied similarly to Helton and Davis (2000) to further investigate the model behavior. The regression sensitivity model explains how much of the overall uncertainty can be explained by the variables during all time periods (measured as R^2). High R^2 often means that a basic linear regression model is a good sensitivity analysis procedure for the case (i.e. there is no significant nonlinearity, non-monotonicity, or interacting or higher order terms). Comparison with a rank-correlated regression R^2 illustrates if there is non-linearity in the model. Second order coefficients should be tested for the most significant uncertainties if they are assumed to be correlated with each other or if the constructed cash flow model is known to have such structures. Also non-monotonic patterns may be tested, yet Samuelsson's proof (1965) and central limit's theorem suggest that the expected value is a random walk regardless of the non-monotonic (e.g. cyclical) pattern of the variable.

As a result, a decision maker is able to model and understand the behavior of the consolidated stochastic process over time and how different cash flow components affect it. Furthermore, the results of this regression sensitivity analysis are of significant use in constructing a forward-looking regression-based value estimator, or a response surface method estimator, for the underlying asset and its stochastic process. As a result, a change in a direct primary source uncertainty variable can affect simultaneously several indirect secondary variables – the underlying asset value and the volatility during different time periods - via regression based estimators. Therefore, by changing a single or even several primary parameter values of the cash flow calculation, a decision maker recognizes immediately how this changes the real option value.

2. References

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